Time-domain reduced-order modelling of linear finite-element eddy-current problems via RL-ladder circuits

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Summary. This paper deals with the lumped-parameter modelling of magnetically linear eddy-current devices which have one terminal voltage and current. The device is first characterised by means of frequency-domain finite-element (FE) computations considering the relevant frequency band, for subsequently fitting constant-coefficient RL ladder circuits of various extent (i.e. number of branches). The accuracy of the ladder-circuit model is assessed in both frequency and time domain. This is successfully applied to the axisymmetric magnetic levitation device of TEAM Workshop problem 28.

1 Introduction

The time-stepping FE simulation of an electromagnetic device may be the suitable approach for studying (and possibly improving) its performance in simple steady-state electrical and mechanical conditions. If the next stage in the analysis/design is about more realistic transient operation, possibly with (closed-loop) control and integration in a wider system, the FE model may become prohibitively expensive. A computationally cheaper model, generically referred to as reduced-order model (ROM), should be derived, inevitably at the expense of a certain loss of accuracy.

Purely mathematical ROMs in various engineering disciplines are based on concepts such as singular value decomposition and snapshot selection [1], and mostly involve little or no physical insight of the device or problem at hand.

Alternatively, ROMs can also be based on the identification of the device via a series of simple static or dynamic FE computations [2], rather than on a manipulation of the FE matrices. Such an approach is more or less straightforward and feasible depending on the characteristics of the electromagnetic device: the number of independent currents (or voltages) n_{i}, the absence/presence of saturable magnetic materials and induced currents, and the number (0, 1 or more) of position degrees of freedom n_{p}. Different particular cases can be distinguished. If the system is magnetically linear without eddy currents, the current-independent $n_{\mathrm{i}} \times n_{\mathrm{i}}$ inductance matrix can be straightforwardly obtained by n_{i} magnetostatic computations. Through tabulation and interpolation of the inductance values, a minimum-cost model of the device is easily achieved. The situation changes drastically in presence of saturable material and/or eddy currents.

In this paper we consider the levitation device of TEAM Workshop problem 28 (TWP28) [3], a relatively simple, though far from trivial case. It consists of a magnetically linear device with one terminal current ($n_{\mathrm{i}} = 1$).

2 Frequency-domain identification

The axisymmetric device of TWP28 comprises two anti-series-connected coils and an aluminium circular disk at height z_{pl} above the coils. Part of the cross-section can be seen in Fig. 1. The well-known magnetic vector potential formulation, in terms of its tangential component a_{ϕ}, is adopted, either in the time or frequency domain. Phasors are denoted by underlined symbols; f and $\omega = 2\pi f$ are the frequency and the pulsation, and $\jmath$ is the imaginary unit.

The instantaneous terminal voltage $v(t)$, current $i(t)$ and associated flux-linkage $\psi(t)$ are linked as:

$$v(t) \;=\; R_0\, i(t) + \frac{\mathrm{d}\psi}{\mathrm{d}t}\,, \tag{1}$$

where $R_0 = 6.73\,\Omega$ is the DC resistance. In absence of eddy currents in the plate, in the low-frequency limit $f \to 0$, we have $\psi = L_0 i$, where $L_0 = 73.2\,\mathrm{mH}$ is the (z_{pl}-independent) DC inductance.

In the frequency domain, in presence of eddy currents in the plate, (1) becomes $\underline{V} = \underline{Z}\,\underline{I}$, where the complex terminal impedance $\underline{Z} = R + \jmath\omega L$ depends on both frequency f and position z_{pl}. The AC resistance $R(f, z_{\mathrm{pl}})$ and inductance $L(f, z_{\mathrm{pl}})$ can be obtained from the FE model via: 1) terminal $\underline{V}$ and $\underline{I}$; 2) the flux-linkage, $\underline{\Psi}/\underline{I} = L + \jmath(R - R_0)/\omega$; or 3) the integral of the magnetic energy and Joule loss density.

The relative change in resistance and inductance, i.e. $\Delta R/R_0$ and $\Delta L/L_0$, with $\Delta R = R - R_0$ and $\Delta L = L - L_0$, is shown in Fig. 3, with a suitable double logarithmic scale, in the 10 to 1000 Hz band, and for three positions, viz $z_{\mathrm{pl}} = 3, 10$ and $17\,\mathrm{mm}$.

3 Ladder-circuit approximation

The frequency-dependent resistance and inductance can be approximately effected with a ladder circuit as

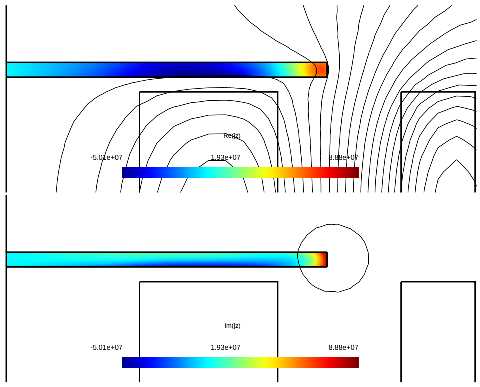

Fig. 1. Flux lines and induced current density in the plate at $f = 1\,\mathrm{kHz}$ and $z_{\mathrm{pl}} = 3\,\mathrm{mm}$, in phase (up) and quadrature (down) with imposed 20 $\mathrm{A_{peak}}$ current

in Fig. 2, with e.g. two additional loops ($n_{\mathrm{b}} = 2$) and two auxiliary (loop) currents i_1 and i_2 besides the terminal current $i(t)$. The $n_{\mathrm{b}}+1$ circuit equations can be written in matrix notation in terms of the column vector $[I(t)]^T = [i(t)\ i_1(t)\ i_2(t)\ \ldots]^T$ and corresponding voltage vector $[V(t)]^T = [v(t)\ 0\ 0\ \ldots]^T$:

$$[V(t)] \;=\; [R]\,[I(t)] + [L]\,\frac{d}{dt}[I]\,, \qquad (2)$$

where $[R]$ is diagonal and $[L]$ tridiagonal. With $n_{\mathrm{b}} = 2$: $[R] = \mathrm{diag}(R_0, R_1, R_2)$ and

$$[L] \;=\; \begin{bmatrix} L_0 & -L_0 & 0 \\ -L_0 & L_0 + L_1 & -L_1 \\ 0 & -L_1 & L_1 + L_2 \end{bmatrix}\,. \qquad (3)$$

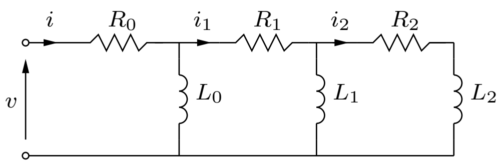

Fig. 2. Ladder circuit with two auxiliary loops ($n_{\mathrm{b}} = 2$)

For a given n_{b} and position, the parameters $R_k(z_{\mathrm{pl}})$ and $L_k(z_{\mathrm{pl}})$ are determined by fitting the ensuing impedance $\underline{Z}_{n_{\mathrm{b}}}(f, z_{\mathrm{pl}})$ to the reference FE impedance $\underline{Z}_{\mathrm{FE}}(f, z_{\mathrm{pl}})$ in the relevant frequency band, e.g. by means of the Nelder-Mead simplex method (nonlinear minimization). Some results are depicted in Fig. 3 with $n_{\mathrm{b}} = 1$ and $n_{\mathrm{b}} = 3$. One observes an excellent agreement with the FE results for $n_{\mathrm{b}} = 3$.

Next a time-domain computation with imposed $f = 1\,\mathrm{kHz}$, 500 $\mathrm{V_{peak}}$ voltage, with z_{pl} =10 mm, is carried out. The Joule losses in the plate follow from the ladder-circuit model by summing $R_k\, i_k^2(t)$ in all resistances but the DC one ($1 \le k \le n_{\mathrm{b}}$). Excellent

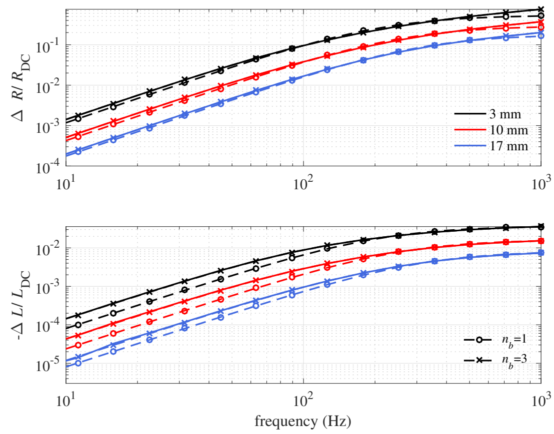

Fig. 3. Relative increase/decrease of AC resistance and inductance with frequency (for 3 different positions), obtained with FE model (full lines) and ladder circuit (markers, $n_{\mathrm{b}} = 1$ and $n_{\mathrm{b}} = 3$)

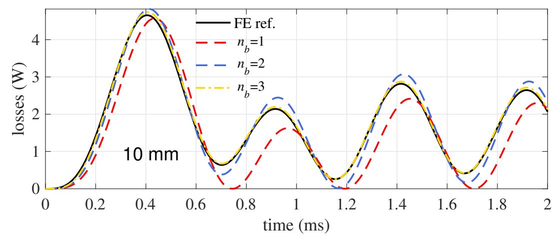

Fig. 4. Losses in the plate versus time ($z_{\mathrm{pl}} = 10\,\mathrm{mm}$; 1 kHz voltage supply), computed with FE and LR-ladder circuits (n_{b} equal to 1, 2 and 3)

convergence towards the FE results is observed with increasing n_{b}.

Further details on the implementation and more results will be given in the full paper. Particular attention will be paid to the coefficient behaviour in terms of the position, what matters for further consideration of the mechanical equation.

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